Blue Screen of Death? Obsolescence and Structural Change in the Computer Age

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Abstract

The rising capital income share and declining productivity growth have been well-documented facts of recent years. This paper argues that one of the forces behind these trends is the evidence-based high obsolescence rates of computer technologies, namely hardware and software, widely used in many types of machines. Using an endogenous growth model calibrated to U.S. data, I show that faster technological obsolescence of machines could be responsible for roughly 30 percent of the observed decline in the productivity growth and all of the observed increase in the capital share. The results suggest that the short technology life-cycle reduces not only the efficiency of capital but also the efficiency of labor indirectly via labor-capital complementarity in final output production, resulting in a decline in the long-run productivity growth. In addition, scarce resources are increasingly allocated to capital augmentation to recoup tech obsolescence. Therefore, the share of income going to capital increases.

Keywords: Technological Obsolescence, Productivity Growth, Capital Income Share
JEL Classification: E25, O30, O41

1. Introduction

The U.S. Economy experienced an exceptional productivity speedup in the mid-1990s. However, over the past 15 years, we have witnessed a noticeable slowdown. Meanwhile, the share of income going to capital has increased, spoiling Kaldor's (1961) fact about constant factor income shares. In this paper, I discuss that the recent trends likely reflect the structural changes characterized by the specific features of the Computer Age. To this end, by drawing on a micro-founded endogenous growth model of the U.S. Economy, I argue that the high obsolescence rate of computer technologies is one of the driving forces behind these trends.

It is hard to pinpoint when the Computer Age started, but the consensual view (among technology experts and historians) is that it was around the 1970s.[1] However, the 1990s were the time of path-breaking innovations in terms of the computers. In particular, the sudden rush to high technologies by the firms occurred around the mid-1990s (Kozmetsky and Yue, 2005), which has led to the use of microprocessors inside many types of machines other than desktops, PCs, etc. to provide *smart* functionality. Labor productivity boomed in the United States during this period (Figure 1 Panel A), thanks to the contributions of computer technologies.

This productivity boom was transitory, but the recent slowdown is structural and predates the Great Recession (Jorgenson, Ho, and Samuels, 2014; Fernald et al., 2017).[2] There are several explanations regarding the underlying reason(s) for this trend. The most popular ones are the declining knowledge diffusion and the increasing market power of large firms. Decker et al. (2016) show that the number of start-ups has fallen, and the job creation has slowed across all sectors of the U.S. since the 2000s, which has led to a failure in transmitting the gains from technological advances into aggregate productivity fully. Akcigit and Ates (2019) and Aghion et al. (2019b) recently argue that the slowing business dynamism mainly results from a concentration of valuable knowledge in the hands of tech giants. Liu, Mian, and Sufi (2022), by the same token, discuss that the decreasing interest rates discourage the competition and thus cause a decline in productivity growth. Some other studies point out that the U.S. Economy has been tending to a lower growth rate for some

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1. The Computer Age is also known as the Information Age.
2. Feldstein (2017) points out that the measures of productivity growth may be inaccurate because of the difficulty of measuring quality changes and the failure of incorporating the value of new products fully. Syverson (2017) and Aghion et al. (2019b) reject this claim and conclude that the slowdown reflects a real change.

time due to declining research productivity (Gordon, 2016; Bloom et al., 2020). On the other hand, Fernald (2015) and Byrne, Fernald, and Reinsdorf (2016) suggest that the slowdown may be just flip side of the all-at-once increase in aggregate productivity in the mid-1990s.
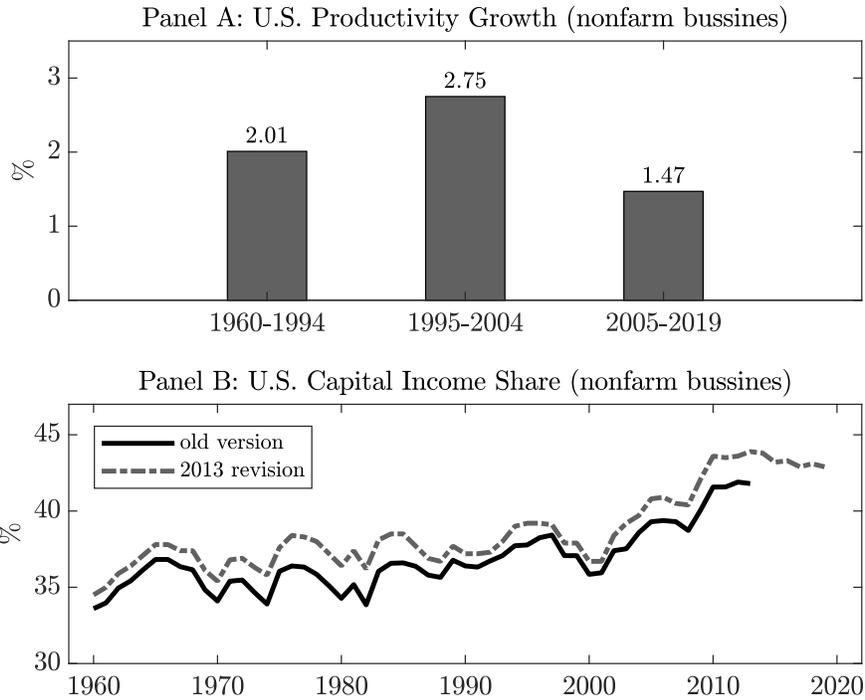
Panel A: U.S. Productivity Growth (nonfarm bussines)

Panel B: U.S. Capital Income Share (nonfarm bussines)

Figure 1: Labor Productivity Growth and Capital Income Share

Source: U.S. Bureau of Labor Statistics (BLS) and the author's calculation

Figure 1 (Panel B) shows the recent evolution of the capital income share in the U.S. nonfarm business sector. Bureau of Labor Statistics (BLS) revised the factor shares in 2013 by incorporating part of intangible expenditures into value-added. Both versions show a steeper rise after 2000. Increasing share of capital income has been reported and discussed so far by many researchers including Elsby, Hobijn, and Şahin (2013), Piketty and Zucman (2014) and Rognlie (2016). Many possible explanations have been offered in the literature, but I will discuss the most closely related ones to this paper.

One suggestion is that automation of the labor tasks by robotics, artificial intelligence, and other forms of high tech may lead to a rise in capital income share (e.g., Fernald

and Jones, 2014; Acemoglu and Restrepo, 2018; Autor and Salomons, 2018). Some works relate the change in factor shares to the rising markups (e.g., Aghion et al., 2019a; Autor et al., 2020; De Loecker, Eeckhout, and Unger, 2020) while the others stress the growing wedge between wage and productivity due to a decline in labor's bargaining power (Bivens et al., 2014; Dube et al., 2020). On the other hand, Karabarbounis and Neiman (2014) point out the declining relative price of investment goods due to the high-tech revolution, which urges firms to use more capital and less labor.

The literature highlights that the changes in productivity growth and capital income share are probably linked and associated to the Computer Age, but for different reasons. This paper, however, explores a unified mechanism that might have led to these trends in the United States. To explain it, I start by asking what makes the computers so specific. However, before addressing this question, let me emphasize again that the computers here refer to hardware and software which are being embedded into many products and tools, including smart machines.

The computers, like all other technologies, are subject to obsolescence, but unlike any other, their obsolescence is very rapid (Oliner et al., 1994; Doms et al., 2004). One of the determinants is the exponential increase in information processing power, as Moore's Law has predicted so far. This situation induces the replacement of microprocessors long before their useful life is over (Geske, Ramey, and Shapiro, 2007; Oliner, 2007). However, the most critical determinant is the strong interdependence between computer components which contributes to systemic obsolescence.[3] In other words, replacement of a component requires replacement of the rest; otherwise, the entire system becomes technically inferior. By this characteristic, the computers differ from the technologies of the past 150 years (Baldwin and Clark, 2000). The lack of compatibility between old and new components also accelerates the obsolescence of complementary technologies (Baldwin and Clark, 1995). Moreover, systemic obsolescence may adversely affect the efficiency of the workers who work with computer-based equipment – through skill-erosion.[4]

I attempt to investigate the macroeconomic implications of this phenomenon through a two-R&D-sector endogenous growth model à la Acemoglu (2003). In the model, there are two types of technological designs (or ideas) created with research effort – H-type

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3. For example, using the latest version of an operating system requires a more powerful central processing unit (CPU). A new CPU can only work with a larger random access memory (RAM). However, if the larger RAM has incompatible settings with the motherboard, it will be necessary to have an upgraded motherboard.

4. See Krusell et al. (2000) for capital-skill complementarity.

and Z-type. Consider that final output is a function of efficient labor, HL, and efficient capital, ZK, such that $F(HL; ZK)$. An increase in H and/or Z implies labor- and/or capital-augmenting technological progress, where profit-maximizing firms determine the form of progress. I relax the baseline framework by imposing the following assumptions: (i) hardware and software are in the category of capital-augmenting technologies; (ii) these types of technologies are subject to a constant rate of obsolescence; hence, in the long run, net capital-augmenting technological progress is nil, and productivity growth is *purely labor-augmenting*; (iii) raw labor resource is homogeneous in skill and quality, and production and R&D sectors compete for labor; (iv) efficient labor and efficient capital are gross complements.

I assume that the economy was shocked in 1995 by an upward shift in the obsolescence rate of capital-augmenting technologies. I calibrate the magnitude of the shock by matching the annual decline in efficient capital-labor ratio of U.S. total industry. Labor productivity booms on impact during the transition due to capital deepening. Then, as observed empirically, productivity growth cools off after around ten years, converging to a lower balanced growth path.

As a result of faster tech obsolescence, capital becomes relatively abundant in the long run. In contrast, efficient capital becomes relatively scarce, which increases its price and thus the relative profitability of investing in capital-augmenting technology designs. This obsolescence-offsetting mechanism yields two interrelated outcomes. First, due to the complementarity between efficient labor and efficient capital, the price effect suppresses the market effect; thereby, capital income share increases and converges to a higher steady-state. Second, a portion of productive resources to be used to produce goods and create labor-augmenting ideas is now devoted to capital-augmenting R&D that has no contribution to productivity growth in the long run. In short, faster obsolescence also slows down the creation of new labor tasks, which ultimately means a decline in the long-run economic growth (and an increase in the capital share).

The permanent obsolescence shock explains roughly one-third of the decline in U.S. productivity growth and all of the increase in the capital share after the mid-2000s. The model outputs also speak to some recent paradoxical patterns of the U.S. Economy, such as low growth rate despite high R&D intensity, weak business investment despite the low rate of interest, and widened gap between wage and productivity. Last but not least, the study shows that the increase in the capital income share is accompanied by the concentration of capital among a limited number of workplaces.

The results are qualitatively robust to alternative calibration scenarios. Consequently, this paper's contribution is twofold. First, I reintroduce tech obsolescence as a new driver by quantifying its potential macroeconomic effects during the high-tech transition and in the long run. Second, I show that faster obsolescence can indeed be a common force behind some notable structural changes. In that sense, this paper identifies a unified mechanism that may allow future empirical investigation.

The rest of the paper proceeds as follows. Section 2 constructs the model and defines the balanced growth path properties. Section 3 calibrates the initial balanced growth path of the model to the U.S. Economy. Section 4 simulates the transition path of the model economy and compares the simulation results with the recent empirical regularities. In Section 5, I perform a comparative statics analysis to provide a broader view of equilibrium responses to generalized exogenous variation in the obsolescence rate. Section 6 covers the concluding remarks.

2. Model

2.1. Preferences and Technology

Let me suppose that the economy is populated with a representative household endowed with a constant unit of labor resource L which can be used in the production of goods or creation of new technology designs. The household also owns capital stock.

Consider that this household has constant relative risk aversion (CRRA) preferences over consumption of final good:

$$U = \int_0^\infty \frac{C(t)^{1-\sigma}-1}{1-\sigma} e^{-\rho t} dt \tag{1}$$

where inverse of σ represents willingness to substitute consumption C between current and future periods and ρ is constant rate of time preference.

The flow budget constraint of the household is

$$\dot{K} + C - rK + wL = Y \tag{2}$$

where r is rental rate of capital and w is wage rate. I assume that there is no capital de-

preciation. Final good, Y, is a constant elasticity of substitution (CES)-type combination of labor-intensive and capital-intensive aggregate goods, Y_L and Y_K, such as

$$Y = \quad Y_L + (1 \quad)Y_k \quad {}^{\frac{1}{}} \tag{3}$$

with $\quad < 0$ and $\quad 2\ (0;1)$, where $\quad$ and $\quad$ denote CES and distribution parameters, respectively. Y_L and Y_L are Dixit-Stiglitz aggregations of labor-intensive and capital-intensive intermediates y_l and y_k, indexed by $\quad$ in the interval $[0; h]$ and $[0; z]$, where h and z are index measures of product variety of the respective type of intermediates.

$$Y_L = \quad \int_{0}^{Z\ h} (y_l(\ ))\ d \quad {}^{\frac{1}{}} \quad \text{and } Y_K = \quad \int_{0}^{Z\ z} (y_k(\ ))\ d \quad {}^{\frac{1}{}} \tag{4}$$

where $\quad$ and $\quad$ represent the substitutability between varieties of the regarding type, $\quad 2\ (0; 1)$ and $\quad 2\ (0; 1)$.

The intermediate goods sector consists of monopolistic firms. To become an intermediate good producer, one must first acquire an entirely new technology design from R&D sector. When this happens, the producer receives a perpetual patent protection over its design and becomes a monopolist with respect to its own differentiated intermediate product.

A labor-intensive intermediate, y_l, is produced with linear production technology by using labor; likewise, capital-intensive intermediate, y_k, is produced using capital.

$$y_l(\ ) = l(\ )\ \text{and}\ y_k(\ ) = k(\ ) \tag{5}$$

R&D sector is the crucial source of technological progress, where the creation of new designs expands the product varieties of labor-intensive and capital-intensive intermediates; h and z. There is free entry in R&D sector. The production rate of new designs is proportional to the amount of labor supplied by the representative household and the existing stock of knowledge from the past research.

R&D-based labor- and capital-augmenting technology equations are defined by

$$\dot{h} = h(b_h L_h) \quad \text{and} \quad \dot{z} = z(b_z L_z \quad) \tag{6}$$

where L_h and L_z represent research effort (unit of working time) devoted to the respective

sector, b_h and b_z denote productivity parameters with $b_h > 0$ and $b_z > 0$, is technological obsolescence rate at which capital-augmenting designs are forcefully retired, and 0.

Here, captures the lack of compatibility between old and new designs. When > 0, the incompatibility would decrease overall knowledge spillover in R&D sector.

2.2. Equilibrium

The equilibrium is time paths of consumption, stock of capital, stock of labor- and capital-augmenting technological knowledge, and resource employment in production and R&D sectors, C, K, h, z L, L_h, L_z, labor- and capital-intensive intermediates and aggregate goods, $\{y_l( )\}_{2h}$, $\{y_k( )\}_{2z}$, Y_L, Y_K, and the prices, r, w, $\{p_l( )\}_{2h}$, $\{p_k( )\}_{2z}$, P_L, P_K, V_l, V_k, such that the household allocates consumption according to the Euler equation and budget constraint, final good producers and intermediate good monopolists maximize their profits; resource allocation is determined by free entry into R&D sector, and all markets are clear.

The solution of the household's optimization problem is characterized by the Euler Equation under no-Ponzi game condition and given initial capital stock $K(0) > 0$.

$$\frac{\dot{C}}{C} = \frac{r - }{} \tag{7}$$

Solving the household's optimization sub-problem yields the relative price of capital-intensive aggregate goods,

$$\frac{P_K}{P_L} = \frac{1 - }{} \left(\frac{Y_K}{Y_L} \right)^{1} \tag{8}$$

I normalize the price of final good to 1. Given perfect competition, final goods producers earn zero profit, hence demand functions for the intermediate goods y_l and y_k of th variety are defined as follows:

$$y_l( ) = Y_L \left(\frac{p_l( )}{P_L} \right)^{\frac{1}{-1}} \quad \text{and} \quad y_k( ) = Y_K \left(\frac{p_k( )}{P_K} \right)^{\frac{1}{-1}} \tag{9}$$

The value function of a labor-intensive monopolist who owns a technology design is

$$V_l(\omega; t) = \max_{[y_l(\omega; s), p_l(\omega; s)]_{s=t}^{\infty}} \int_t^{\infty} \exp\left(-\int_t^s r(s)\, ds\right) \pi_l(\omega; s)\, ds \tag{10}$$

Analogously, for a capital-intensive monopolist, it is[5]

$$V_k(\omega; t) = \max_{[y_k(\omega; s), p_k(\omega; s)]_{s=t}^{\infty}} \int_t^{\infty} \exp\left(-\int_t^s (r(s) + \delta)\, ds\right) \pi_k(\omega; s)\, ds \tag{11}$$

where instantaneous profits of two types of monopolists for all ω and t are

$$\pi_l(\omega; t) = y_l(\omega; t)(p_l(\omega; t) - w) \quad \text{and} \quad \pi_k(\omega; t) = y_k(\omega; t)(p_k(\omega; t) - r) \tag{12}$$

Marginal costs of production of labor- and capital-intensive intermediates are, respectively, w and r. Given the demand functions (9), monopoly price is a constant markup over marginal cost.

$$p_l(\omega) = \frac{w}{\alpha} \quad \text{and} \quad p_k(\omega) = \frac{r}{\alpha} \tag{13}$$

Since the optimal solution is same for all monopolists (of the regarding type of technology), price of all intermediates is same: $p_l(\omega) = p_l$ and $p_k(\omega) = p_k$. Thus, the symmetric equilibrium implies that $y_l(\omega) = y_l$ and $y_k(\omega) = y_k$.

Then, using (5), I define the market clearing condition for labor and capital in production sector,

$$\int_0^{Z_h} l(\omega)\, d\omega = L \quad \text{and} \quad \int_0^{Z_z} k(\omega)\, d\omega = K \tag{14}$$

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5. Note that the obsolescence rate appears in (12) because sum of all discounted future profits is same for all $\omega \in [0; z]$.

Aggregating across all intermediate producers of the respective technology type, I obtain total profits of labor- and capital-intensive intermediate sectors as follows:

$$\pi_L = \frac{1}{\quad} wL \quad \text{and} \quad \pi_K = \frac{1}{\quad} rK \tag{15}$$

where $\pi_L \quad h_l$ and $\pi_K \quad z_k$. From (5) and (14) I already have production level of intermediates, $y_l = \frac{l}{h}$ and $y_k = \frac{K}{z}$. Substituting these into (4) yields total supply of labor- and capital-intensive aggregate goods, i.e., total supply of efficient labor and efficient capital,

$$Y_L = HL \quad \text{and} \quad Y_K = ZK \tag{16}$$

where $H \quad h^{1-}$ and $Z \quad z^{1-}$. Let me redefine the relative price of capital-intensive aggregate goods (or efficient capital) such as

$$P \quad \frac{P_K}{P_L} = \frac{1}{\quad} \left(\frac{ZK}{HL}\right)^{1} \tag{17}$$

Since factors market is competitive, labor and capital are paid their marginal products. Hence, wage and rental rates are

$$w = \quad HP_L \quad \text{and} \quad r = \quad ZP_K \tag{18}$$

Let denote efficient capital-labor ratio, $\frac{ZK}{HL}$. Using (16) and (17), I obtain the relative share of capital in income,

$$\frac{P_K Y_K}{P_L Y_L} = \frac{1}{\quad} \tag{19}$$

Total remuneration in labor-intensive sector equals $P_L Y_L = \frac{wL}{\quad}$, which is labor income (including profits) accrued to this sector; just as, in capital-intensive sector, the total remuneration equals $P_K Y_K = \frac{rK}{\quad}$.

9

R&D and production sectors compete for labor. Profit maximization problem of R&D sector (the first problem is for labor-augmenting and the second is for capital-augmenting), then, is

$$\max_{\{L_h\}} \ V_l b_h h \ - \ w \ L_h \ \text{ and } \ \max_{\{L_z\}} \ V_k b_z z \ - \ w \ L_z \tag{20}$$

V_l and V_k are, like those mentioned earlier, net present discounted value of a monopolist who acquires a new design to produce its differentiated intermediate; in this regard, the innovators are ex-post monopolists. However, ex-post rents are just enough to cover the cost of entry, which implies that the profit maximization problem of the R&D sector is static. Hence, there is free-entry in R&D sector, which necessitates the following condition:

$$V_l b_h h \ = \ V_k b_z z \ = \ w \tag{21}$$

Note that the capital-augmenting R&D sector does not incorporate technological obsolescence – a dynamic externality – into the profit maximization problem since there is no dynamic monopoly rent in R&D sector.

Finally, labor devoted to production and R&D is identical in skill and quality. Then, labor market clearing condition requires

$$L + L_h + L_z \ - \ L = 1 \tag{22}$$

where I normalize total labor input (L) to 1.

2.3. Balanced Growth Path Properties

The model economy remains on a balanced growth path (BGP) only if technological change is purely labor-augmenting (Uzawa, 1961), which immediately implies that in the long run, net capital-augmenting technological change is zero, $\frac{\dot{z}}{z} = 0$. Therefore, the stock of capital-augmenting technological designs, z, and the price of efficient capital, P_K, are constant.[6] However, there has to be some research effort devoted to creating new designs

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6. From the Euler equation (7), interest rate r must be constant along balanced growth trajectory. Proof by contradiction: Assume that a BGP has capital-augmenting technological progress. Then, (18) implies that the price of capital-intensive goods decreases unboundedly to keep interest rate constant, i.e., for $r = \bar{r}$, $\lim_{t \to \infty} z(t) = \infty \Rightarrow \lim_{t \to \infty} P_K = \infty$; hence, there would be no incentive to produce capital-

to keep the technology level unchanged given the obsolescence, $L_z = \frac{}{b_z}$.

The price of efficient labor, P_L, also is constant because, along a BGP, wage and productivity grow at the same rate. Differentiating the first equation in (18) and using labor-augmenting technology equation in (6), BGP employment in labor-augmenting R&D sector is defined by $L_h = g \frac{}{b_h(1-)}$, where g is BGP (labor) productivity growth.

The values of labor- and capital-augmenting designs are as follows:

$$V_l = \frac{1}{r-\tilde g} \frac{wL=h}{} \quad \text{and} \quad V_k = \frac{1}{r+} \frac{rK=z}{g} \tag{23}$$

where $\tilde{} \quad \frac{1-2}{1}$. From (21) and (23), this time I obtain BGP employment in production sector as $L = (r - \tilde g) \frac{}{b_h(1-)}$.

Recall that labor market clearing condition is $L + L_h + L_z = 1$. Plugging long-run employment shares into the equilibrium condition and then rearranging it, I can write BGP productivity growth as below:

$$g = \frac{1}{} \frac{b_h - \frac{b_h}{b_z} - \frac{1}{}}{\tilde{} + 1} \tag{24}$$

From Euler Equation (7), BGP interest rate must be given by

$$r = g_C + \tag{25}$$

Consumption grows at the same rate as productivity and total output, $g = g_C = g_Y$, given the equilibrium interest rate.

In the equilibrium, the entrepreneurs are indifferent between creating labor- and capital-augmenting technology designs. Hence, using 19, (21), and (23), I retrieve BGP relative capital income share,

$$\frac{1}{}(\ ) = \frac{g(\ -1) + \ +}{g(\ -\tilde{}) +} \tag{26}$$

where $\frac{1}{1-} \frac{b_h}{b_z}$. (26) and (19) imply that Y_L and Y_K must grow at a common rate;

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intensive intermediates, and thus no capital-augmenting technological progress. This result contradicts the first assumption.

11

thereby, the relative price of efficient capital, P, is constant in the long run. The factor income shares remain constant, so does , and capital, K, must grow at the same rate as productivity.

Eventually, a balance-growth path of this economy admits the following condition:

$$g = g_C = g_K = g_Y \tag{27}$$

Note that there are many combinations of efficient capital-labor ratio, , distribution parameter, , and elasticity of substitution parameter, , which ensure a unique BGP. In the coming section, I will normalize initial BGP to 1, so 1 will identify capital income share at a fixed point in the initial steady-state.

3. Calibration of Initial BGP

I calibrate five model parameters that define the initial balanced growth path of the model by matching five moments which are long-run conditions of the U.S. Economy before the mid-1990s, as listed in Table 1. This calibration exercise does not reflect the patterns of the last 25 years, which I aim to simulate in the next section.

The model parameters in concern are substitutability between labor-intensive intermediates ( ), productivity in labor-augmenting R&D sector (b_h), productivity in capital-augmenting R&D sector (b_z), obsolescence rate of capital-augmenting technologies ( ), and rate of time preference ( ). CES parameter, , is from Klump, McAdam, and Willman (2007) and EIS parameter, , is from Hall (2009).[7] The parameter is for substitutability between capital-intensive intermediates, and I take it from Lindenberg and Ross (1981).[8]

The rate of time preference is mostly ranging from 0.02 to 0.04 in the literature. The calibrated value slightly lies outside this range but not unusually low. The empirical literature does not provide much guidance for substitutability between labor-intensive intermediates, . However, by the model construction, it should be higher than the assigned value of substitutability between capital-intensive intermediates, . Productivity

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7. Klump, McAdam, and Willman (2007) estimate elasticity of substitution between 0.5 – 0.65 for nonresidential private sector over 1953-1998. See Section 5 for the sensitivity of model outputs to choice of .

8. Lindenberg and Ross calculate Tobin's q for different industrial sectors for every two years from 1960 to 1976, which equals to markup parameter in case of constant returns to scale. I take the simple average of the calculated q ratios for electric and nonelectric machinery industries (SIC 35-36) in 1976.

in labor- and capital-augmenting R&D sectors, b_h and b_z, are between 0 — 1 as expected.[9] I internally calibrate the rate of technological obsolescence δ as 0.04. This value is within the range that previous applied works widely assume (e.g., Bernstein and Nadiri, 1988).

Table 1: Baseline Calibration of Model Parameters

	Value	Source/Target
Assigned Parameters		
Substitutability between Labor and Capital (σ)	1	$\frac{1}{1-\sigma} = 0.5$ by Klump et al. (2007)
Elasticity of Intertemporal Substitution EIS ($\frac{1}{\theta}$)	1.2	Hall (2009)
Substitutability between Capital-intensive Goods (ϵ)	0.55	Lindenberg and Ross (1981)
Calibrated Parameters		
(1) Substitutability between Labor-intensive Goods (ν)	0.8035	
(2) Productivity in Labor-augmenting R&D (b_h)	0.6676	g^*; L^*, L_z^*;
(3) Productivity in Capital-augmenting R&D (b_z)	0.3128	
(4) Technological Obsolescence Rate (δ)	0.0391	
(5) Rate of Time Preference (ρ)	0.0183	$r^* = \theta g + \rho$
Target Moments (%)		
(1) Productivity Growth (g^*)	2.01	BLS LPC series
(2) Capital Income Share ($1 - \alpha$)	37.1	BLS LPC series
(3) Rate of Interest (r^*)	5.85	Farhi and Gourio (2018)
(4) Employment in Production (L^*)	75	Acemoglu and Autor (2010)
(5) Employment in R&D sectors (L_z^*)	12.5	$L_z = L_h$

Target moments denoted with an asterisk in Table 1 are, respectively, (labor) productivity growth, capital income share, rate of interest, employment in production sector, and employment in capital-augmenting R&D sector. Regarding the recent empirical regularities, investigating the behaviors of productivity growth and capital income share is the central

9. Growiec, McAdam, and Mućk (2018) calibrate b_h and b_z between 0 — 1 using a similar theoretical framework, but much less than the values in Table 1. However, their functional form of factor-augmenting technological progress differs from what I use in this paper.

motivation of this paper. *Interest rate* is the relevant moment that captures preferences and estimated value of a technology design and represents cost of investment. Finally, labor in final production and R&D sectors reflects resource allocation determining which type of innovations from the set of potential ones are carried out. Although these moments do not have direct real-world counterparts, imposing assumptions on them is vital to study the dynamics of the model.

The labor productivity growth is the average annual growth of real output per hour of all persons in the U.S. non-farm business sector over 1960-1994, which I calculate using Labor Productivity and Cost (LPC) series from Bureau of Labor Statistics (BLS).[10] Before taking the average, I remove cyclical components by using Hodrick-Prescott filter with the smoothing parameter $= 6.25$ recommended by Ravn and Uhlig (2002). Capital income share is simply the part of the income which is not going to labor in the U.S. non-farm business sector. I obtain labor income share statistics from BLS.[11] The capital income share moves around 37 percent during the period in concern.[12]

In the choice of interest rate moment, I follow the calibration strategy of Jones and Williams (2000), and accordingly, the moment to be targeted is model-based equity returns (over 1984-2000) from Farhi and Gourio (2018).

For the employment in production sector, I target the share of the (working) population employed in routine (manual and cognitive) and non-routine manual occupations in 1979, categorized by Acemoglu and Autor (2010).

As a caveat, this calibration strategy carries some limitations. The studies adopting the occupation-base approach generally treat the employment in routine occupations as unskilled workers. Yet, in the model, raw labor is uniform in terms of skill and quality. In short, I do not feel comfortable simply calibrating it to 75 percent. Hence, in Section 5, I will allow the initial target, L , to take on a pre-determined range of values and examine the robustness of the results to this range. To calibrate labor resource devoted to R&D, I decrease the parameter dimension by assuming that in initial BGP, two R&D sectors have equal shares of research effort.[13]

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10. I did not take into account the data before 1959 to avoid the post-war expansionary period.

11. Labor income is the sum of employee compensations and proprietors' labor compensation, based on *labor-basis* approach.

12. Distribution parameter   denotes initial BGP labor income share, where I normalize efficient capital-labor ratio to 1; $_0$ so that $Y_0 = Y_{K_0} = Y_{L_0}$ and $_0$. Thereby, factor income shares are independent of CES parameter   in the initial steady-state.

13. This simplification does not significantly affect the equilibrium results.

4. A Faster Technological Obsolescence

I study the medium-run behaviors of key variables outside of BGP as well as the change in equilibrium path in response to a permanent obsolescence shock. I compare the simulation results with some notable empirical facts of the U.S. economy.

I use *relaxation method* for computational solution of the system, relying on the work of Trimborn, Koch, and Steger (2008). Appendix B provides the transformed system of differential equations and technical aspects regarding the computation procedure.[14]

4.1. Transition Dynamics

Suppose that initially, the economy stays in a steady-state (balanced growth path). Recall initial BGP moments that are provided in Table 1, e.g., the labor productivity growth, the rate of interest and the capital income shares, g, r and 1 , where > 0.

Figure 2 illustrates the transition paths of key variables when a permanent obsolescence shock supposedly perturbed the economy in 1995. For the period in interest, I numerically discipline the transitional adjustment by targeting the annual decline in efficient capital-labor ratio, , of 2.3 log points from Lawrence's (2015) calculation.[15] Consequently, goes up by 51.5 percent to match this decline.

The model economy moves away from the initial BGP since an increase in the obsolescence rate of capital-augmenting technologies pushes down the efficiency level of capital, $\frac{Z}{Z} < 0$. This implies that given the level of output, capital becomes abundant, and therefore the rate of interest falls. On impact of the shock, the rate of labor augmentation jumps up along innovation possibilities frontier; thereby, labor productivity bursts.[16] Meanwhile, efficient capital actually becomes scarce relative to efficient labor, $- < 0$, so its relative price, $\frac{P_K}{P_L}$, rises.

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14. See for Patrikalakis and Maekawa (2002) for further technical details.

15. Lawrence calculates the annual decline in the efficient capital-labor ratio over 1999-2010 for total U.S. industry with an estimate $= 0.494$. Note that and CES parameter have intrinsic links as a natural result of CES functional form. Thus, targeting in a calibration exercise may be misleading unless one does have prior information about . For more detail, see Section 5.

16. Note that I define labor productivity growth (during transition) as $\frac{Y_L}{Y_L}$ $\frac{L}{L} = \frac{H}{H}$. On the other hand, if calculated from the final production function such that $\frac{Y}{Y}$ $\frac{L}{L} = \frac{H}{H} + a -$, labor productivity growth will have two sources at short-term horizon: deviation of efficient capital-labor ratio from its long-run trend and labor-augmenting technological change. In this case, the rate of labor augmentation corresponds to a sort of residual growth when the economy is out of BGP. To overcome this conceptual contradiction, I stick to the model's simplest definition of labor productivity growth.
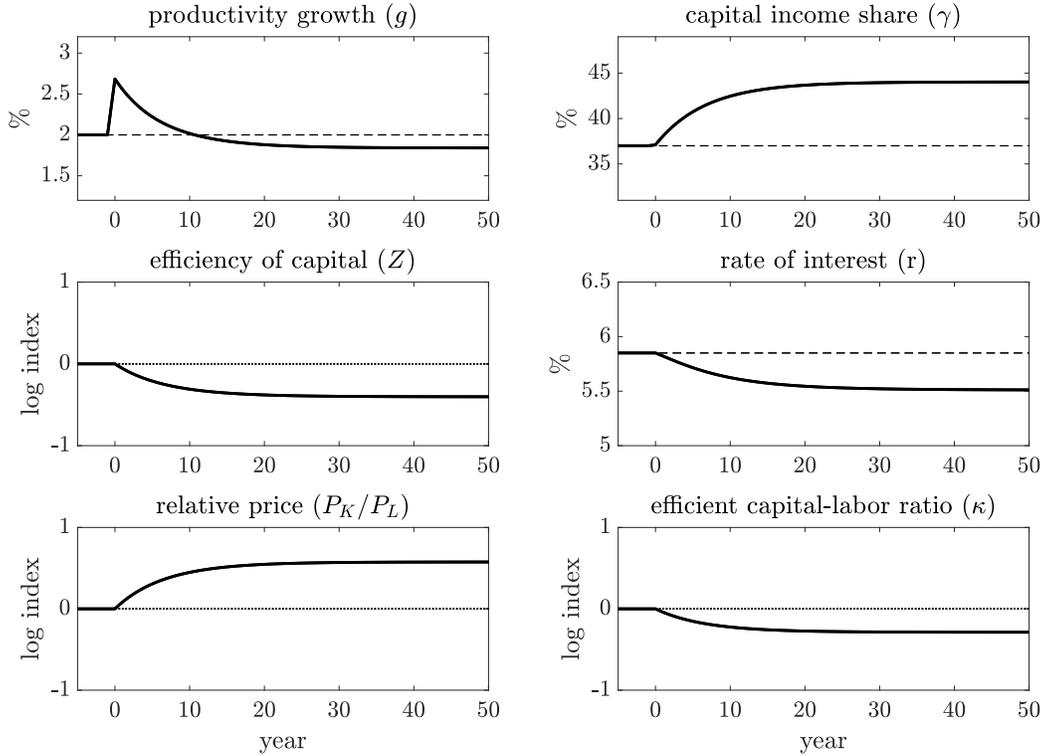
Figure 2: Summary of Transition

Productivity of capital cannot fall steadily, however. The price effect induces capital-augmenting innovations to economize the use of efficient capital that has become increasingly expensive. Therefore, a part of labor resource previously devoted to producing goods and creating labor-augmenting technological ideas moves to capital-intensive sector for R&D. There will be both labor- and capital-augmenting technological change until research effort for capital augmentation comes up to the line with faster obsolescence.

The rate of interest converges to its new BGP level r^*, which is lower than r, and the productivity growth also converges to a lower BGP, $g^* < g$ after a ten-year speedup.

The relative price of efficient capital stays high, and thus the relative profitability of capital-intensive sector, which keeps capital-augmenting investments attractive to offset the high rate of technological obsolescence. Eventually, due to factor complementarity, the increase in relative price (of efficient capital) surpasses the decrease in the efficient

16

capital-labor ratio. Through this channel, the capital income share settles in a higher BGP level, $(1 \quad) > (1 \quad)$.

4.2. Notable Structural Changes

Table 2 compares the model-implied structural changes in untargeted moments with the corresponding ones in the data. The half-life of model productivity growth is around ten years, which coincides with the boom period. Thus, I determine the new BGP period roughly between 2005 and 2019.[17]

Table 2: Comparison of Simulation Results with Some Observed Changes

″ by 51.5%	% point change	
Moments (untargeted)	**Data**	**Model**
(1) Productivity Growth	⇓ 0.54	⇓ 0.17
(2) Capital Income Share	″ 6.3	″ 6.9
(3) Rate of Interest	⇓ 0.96	⇓ 0.34
(4) Employment in Production	⇓ 8.9	⇓ 5.1

Source: (1)-(2) BLS MLSP series and the author's calculation. (3) Farhi and Gourio (2018). (4) Acemoglu and Autor (2010). *Note*: The change in capital income share from 1960-1994 to 2010-2019. The change in rate of interest from 1985-2000 to 2001-2015. The change in labor in production from 1979 to 2007.

U.S. labor productivity (non-farm business sector) has grown by only 1.47 percent annually since 2005, compared with the annual growth rate of 2.01 percent over the period 1960-1994.[18] The 51.5-percent increase in explains about one-third of this decline. Of course, there could be more than one structural source of the slowdown, given demographic changes and stagnant educational attainment.[19] Still, an increase in the rate of technological obsolescence may be one of the forces behind this trend. Likewise, under the calibrated parameters, the obsolescence shock explains around one-third of the fall in the interest rate.[20]

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17. It can go beyond, but due to the COVID effect, I only consider the period until 2019.
18. Productivity series are HP-smoothed based on yearly frequency.
19. See Barro and Lee (2013) for educational attainment statistics.
20. Note that the observed change in this moment stands for the decline in equity returns, so it does

The capital income share began to hike in the late-1990s, and towards the end of the 2010s, it has increased by 6.3 percent points compared with its previous long-run level. The model captures all this increase slightly overshooting.

The employment share in the occupations requiring non-routine cognitive tasks increased by 8.9 percent from 1979 to 2007. I take this figure as a proxy for increase in R&D effort (or decrease in labor devoted to production). The model undershoots the extent of this change, yet it shows that labor resource allocated to creative activities has risen over time.[21] The striking point is that this rise does not generate an extra productivity increase either in the model or the real world. Moreover, we have observed a slowdown in productivity growth.

Table 3: Long-run Changes in Selected Model Variables

Variables	% point change
Employment in Labor-augmenting R&D (L_h)	↓ 1.1
Employment in Capital-augmenting R&D (L_z)	↑ 6.2
	log deviation
Efficient Capital-Output Ratio ($\frac{ZK}{Y}$)	↓ 0.17
Capital-Output Ratio ($\frac{K}{Y}$)	↑ 0.23
Wage-Productivity Gap	↑ 0.23

Note: See Appendix A for the algebraic expression of wage-productivity gap.

Table 3 attempts to clarify the potential mechanism behind this puzzle by showing changes in employment in two R&D sectors separately. Faster obsolescence of capital-augmenting technologies requires more research effort. While some of this need is compensated by the production sector, the rest is dealt by the labor-augmenting R&D sector. Thus, in the long run, where technological progress is purely labor-augmenting, the rate of knowledge creation decelerates with a decrease in L_h, which translates into a slowdown in productivity growth.

not directly reflect the historic fall of risk-free interest rate. On the other hand, Shiller's (2015) cyclically adjusted price-to-earning ratio (CAPE) indicates poor long-term equity returns for the period in concern, which is consistent with the model prediction.

21. I leave the discussion on job polarization and skill premiums to a future study, as the scope of this paper is not oriented toward these.

The simulation results in Table 3 also stress the recent paradoxical behavior of U.S. business investment. The model predicts a higher steady-state capital-output ratio. This outcome is reasonable because a downward change in capital-augmenting trajectory reduces the equilibrium interest rate. However, data suggests that investments have been weak over the last 15 years despite the low cost of investing.[22] Especially, the lacklustre in equipment investment is quite noticeable (Figure C.1).

Equipment growth is a relevant metric for efficient capital accumulation since it broadly incorporates both new and (relatively) old technologies.[23] Taking this into account, the model-simulated downward direction of efficient capital-output ratio, $\frac{ZK}{Y}$, seems consistent with the weak equipment investment. Also, together with a higher $\frac{K}{Y}$, this prediction indicates an abundance of capital equipped by short-lived technologies. If so, relative technology *size* of the past investments might reduce, and the capital formation could shrink in terms of efficiency units. Thus, depending on measure, the decline in efficient capital would manifest itself as a capital shortfall in the data.

Finally, I also want to highlight the decoupling wage rate from productivity, which has been pronounced especially since the mid-1990s (Figure C.2). The model shows a 23-log points deviation from the initial wedge. I can say that this productivity paradox could be linked with the recent dramatic increase in the capital share. In the model, wages growth lags behind labor productivity growth during transition because productivity gain of the boom period (1995-2004) goes to a relatively smaller number of capital-intensive firms, contributing to the gap between labor payments and productivity in the long run.[24]

5. Comparative Statics

In the previous section, I have studied the transition mechanism under the baseline calibration. Here, I provide a broader view of equilibrium responses to various obsolescence shocks under alternative calibration scenarios. I aim to discuss the robustness of qualitative conclusions for different values of model parameters that I externally set. While doing this, I also check the robustness for a range of labor in production sector (L).

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22. Weak U.S. business investment is also documented by Gutiérrez and Philippon (2016) and Farhi and Gourio (2018)

23. Private real fixed investment in equipment series of the U.S. national income and product accounts (NIPAs) includes expenditures on new equipment, and net transactions in used equipment and own-account production of equipment. See Hulten (1991) for capital aggregation from flow investment.

24. The number of capital-intensive firms with differentiated products is lower in the new BGP, $z < z$, given the higher steady-state capital-output ratio.

Scenario I: Let me start with a possible uncertainty resulting from selecting the initial target of labor in production sector, L. In the baseline calibration, I choose it as the share of the working population employed in routine (manual and cognitive) and non-routine manual occupations. This form of categorization commonly treats the population employed in production as unskilled workers.

However, in the model, I do not impose such distinction. So, it is unclear which initial target is correct or reasonable due to the lack of conceptual overlap. Thus, by pre-defining a range for L, I compare the baseline equilibrium response with those for the upper and lower bounds, where the baseline target value .75 is the median of that range.
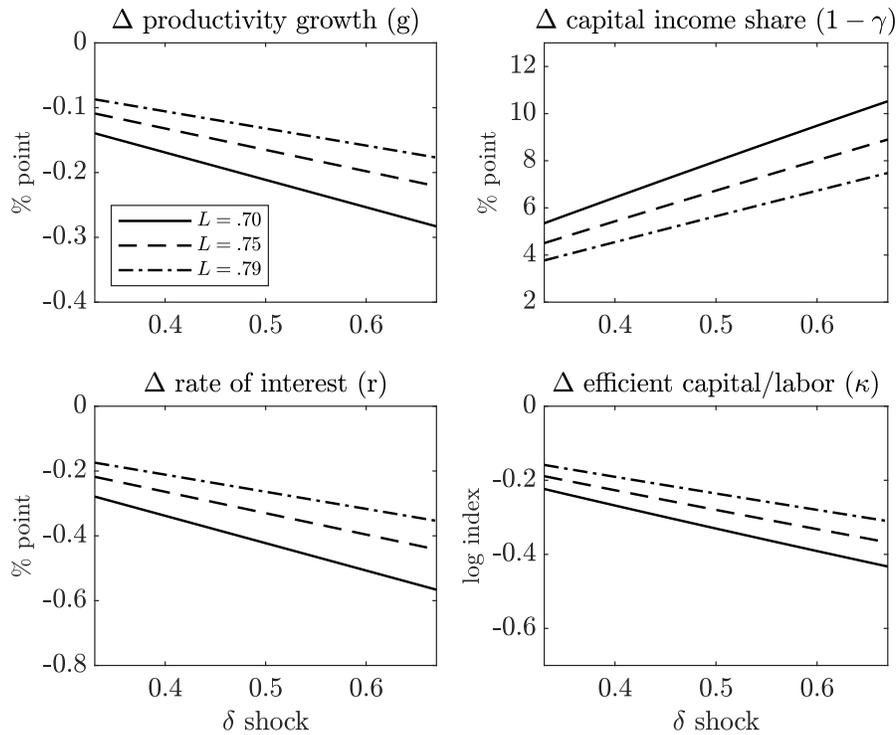

Figure 3: Model Response (Scenario I)

Scenario I: g = 0.0201, (1) = 0.371, r = 0.0585, = 1; = 1, = 2, = 0.55

I register the other initial target moments – productivity growth, capital income share, and rate of interest – while I allow joint-variation of systematically dependent model parameters to match these targets for $L \in \{.70, .75, .79\}$.[25]

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25. Systematically dependent model parameters are substitutability between labor-intensive intermedi-

I plot changes in predicted values of model variables in Figure 3. Consider that the employment in production sector is initially higher in a steady-state. In this case, long-run productivity growth is less responsive to the shock because capital-augmenting R&D sector mainly demand labor from production sector where labor is relatively abundant. Therefore, labor outflow from labor-augmenting R&D sector stays limited compared with the baseline case, which makes growth of wage-rental ratio less sensitive to the shock through the channel of less sensitive productivity growth. As a result, capital income share would not increase as much as in the case of smaller L.

Scenario II: I investigate how the model response changes if   is higher, i.e., the markup factor in capital-intensive sector is lower.[26]
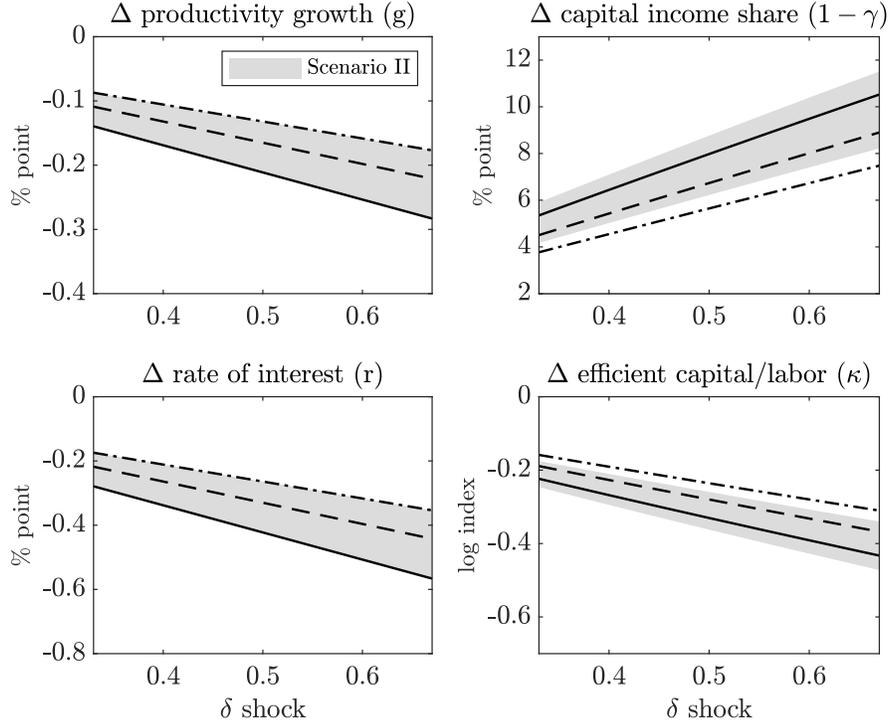

Figure 4: Model Response (Scenario II versus Scenario I)

Scenario II: g = 0:0201, (1   ) = 0:371, r = 0:0585,   = 1;   = 1,   = 2,   = 0:6

Note: Gray areas Scenario II for all L 2 [:70; :79]; black lines Scenario I for L 2 f:70; :75; :79g

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ates,   , productivity parameters in R&D sector, b_h, b_z, and initial technological obsolescence rate. Time preference rate,   , is linearly dependent on EIS parameter,   , given productivity growth and rate of interest. Also, CES production function is initially normalized, that is   = 1.

26. From now on, for an understandable graphical representation, I will show scenario-based model responses for all L 2 [:70; :79], by comparing with Scenario I.

Lindenberg and Ross (1981) measure monopoly power for different sectors as an inverse of Tobin's q in case of constant returns to scale. I take baseline $\quad$ = 0.55, average of q ratios in SIC 35-36 in 1976.[27] Over the period 1960-1976, it is around 0.6 for the same sectors. Thus, I would like to check for a higher value of $\quad$.

Figure 4 shows that the responses of labor productivity growth and interest rate to shock do not change with an alternative calibration of $\quad$. However, capital income share is responsive to the choice of $\quad$ because efficient capital-labor ratio, $\quad$, is more sensitive to the shock. This implies, in turn, given the CES parameter, $\quad$ < 0, a more sensitive relative price of efficient capital. In other words, when the shock hits, the price effect transmits capital's efficiency loss on labor income to a larger extent. Hence, the change in factors income shares would be greater compared with the baseline scenario.

Scenario III: Let me suppose that the representative household is more willing to substitute aggregate consumption over time, i.e., $\quad$ is lower.
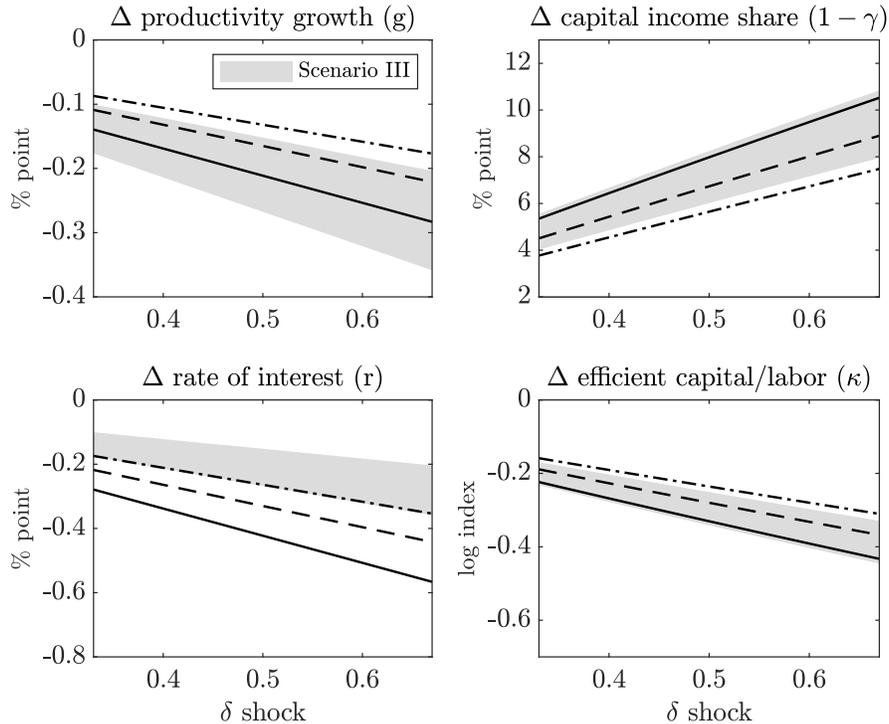

Figure 5: Model Response (Scenario III versus Scenario I)

Scenario III: g = 0.0201, (1 $\quad$) = 0.371, r = 0.0585, $\quad$ = 1; $\quad$ = 1, $\quad$ = 1, $\quad$ = 0.55

Note: Gray areas Scenario III for all $L \in [.70; .79]$; black lines Scenario I for $L \in \{.70; .75; .79\}$

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27. SIC 35 and 36 are Industrial Machinery & Equipment and Electronics & Other Electric Equipment.

Staying in a plausible range of time preference, , given r , I investigate the model's response for the case of = 1; one-for-one relation between consumption growth and rate of interest (Figure 5).

A lower value of strengthens the intertemporal effects in the calibrated model. Therefore, equilibrium productivity growth (consumption growth) is more responsive to shock, whereas equilibrium interest rate is less responsive. On the other hand, has a negligible effect on the sensitivity of capital income share.

Scenario IV: I examine change in model response for an alternative value of capital-labor substitutability degree. Most evidence on CES production functions with factor-augmenting technological bias affirms elasticities range of 0.5 0.7, that labor and capital are gross complements.[28] For Scenario IV, I set elasticity of substitution to 0.6, that is = 0.67.
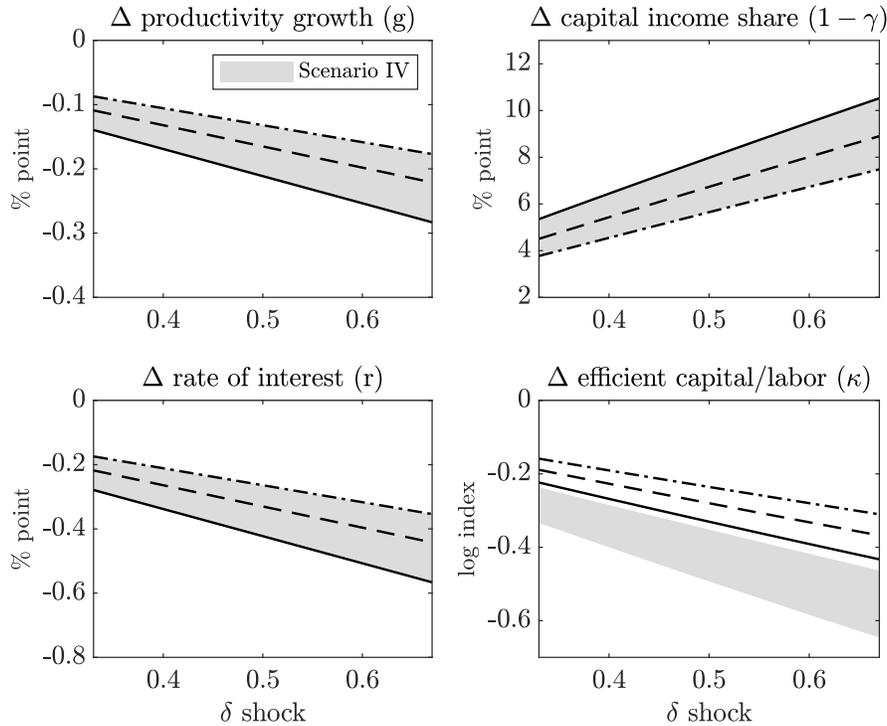

Figure 6: Model Response (Scenario IV versus Scenario I)

Scenario IV: g = 0.0201, (1   ) = 0.371, r = 0.0585, = 1; = 0.67, = 2, = 0.55
Note: Gray areas Scenario IV for all L 2 [.70; .79]; black lines Scenario I for L 2 f.70; .75; .79g

28. See, e.g., Antras (2004), Klump, McAdam, and Willman (2007), León-Ledesma, McAdam, and Willman (2010)

Figure 6 reveals that productivity growth and capital income share are not sensitive to an alternative calibration of CES parameter. This is an expected outcome because, in a balanced growth, both variables are independent of , while underlying efficient capital-labor ratio, . On the other hand,   is very responsive to   shock for a lower (or higher) degree of complementarity because   and   have intrinsic link by CES functional form of final production.

6. Conclusion

I have provided a novel and unified explanation for the secular changes in growth and income distribution that have been recently observed in the U.S. Economy. My study has focused on the evidence-based rapid obsolescence of computer technologies that possibly led to a general increase in the obsolescence rate of all capital-augmenting technologies. I have used an endogenous growth model based on the work of Acemoglu (2003), which provides a convenient theoretical framework to isolate the potential macroeconomic implications of the fast-changing technologies.

In this setting, this paper does multiple things. First, it shows that a high obsolescence rate of capital-augmenting technologies could be responsible for a significant portion of the decline in productivity growth and full extent of the increase in capital income share. Second, it sheds light on the debates over weak investment patterns, and the widened gap between productivity and wage. Third, the study shows that long-run productivity growth may decline even as scarce resources devoted to knowledge creation increase.

The calibrated model predicts a higher research intensity in the long run as faster obsolescence increases the need for more research effort to create new technology designs. However, this extra effort goes down the drain since it is all devoted to capital-augmenting R&D sector in which long-run technological change is nil. The force running in favor of capital here is the higher profitability of capital-augmenting designs relative to labor-augmenting ones because faster obsolescence makes efficient capital relatively scarce and more expensive. Thus, profit-maximizing technology firms start to devote more resources to capital augmentation than before, and they do it not only by absorbing away from production sector but also from labor-augmenting R&D sector. Long-run productivity growth declines in this way.

Based on the simulation results, accumulations of both efficient labor and efficient capital decelerate, but the erosion of the efficient capital stock is relatively more significant. As these factors are gross complements in final output production, producing an additional unit of output requires paying more to efficient capital and less to efficient labor. Consequently, the price effect dominates the market size effect, and the portion of income going to capital increases.

As all this happens, the equilibrium interest falls, and production becomes relatively more capital-intensive. However, the increase in capital per worker fails to yield any deepening effect in the long run due to the concentration of capital in fewer workplaces. Under the general equilibrium conditions, this is the intrinsic consequence of the decline in capital efficiency; more capital is now employed to produce the same level of output.

In the same vein, considering the abundance of capital, one might think that investments must have taken off, but this would be misleading depending on how one measures investment. While statistical institutions aggregate capital stock from flow investment, they give weights to past investments based on their technology size. Only, due to faster tech obsolescence, this size could decrease, and therefore the stock of capital would fall short in effective term. It is quite possible that the observed decline in U.S. business investment could be associated to this.

The study also provides a consistent result with the longest expansion period of the U.S. Economy, which began in the mid-1990s and lasted through the mid-2000s. When the model economy is shocked by the increase in obsolescence rate of capital-augmenting technologies, firms initially choose to augment labor to keep up with this change. Hence, labor productivity booms. However, this speedup is temporary because, as mentioned above, much of the research effort will then be directed toward stopping the ongoing loss in capital efficiency. Moreover, during the boom period, wage grows slower than productivity, so the gap between pay and productivity becomes larger in the long run.

Finally, in the study, I implicitly assume that the obsolescence rate outpaces the arrival rate of new capital-augmenting technology designs. This assumption can be relaxed for a future study under a Schumpeterian framework. On the other hand, the presence or the extent of a general rise in technological obsolescence remains an empirical question. Nonetheless, given the widespread use of hardware and software, it is undeniable that the rapid obsolescence of the computers cannot be innocent in some pervasive changes that have recently happened in frontier economies like the United States.

References

Acemoglu, Daron. 2003. "Labor-and Capital-Augmenting Technical Change." *Journal of the European Economic Association* 1 (1): 1–37. https://doi.org/10.1162/154247603322256756.

Acemoglu, Daron, and David Autor. 2010. *Skills, Tasks and Technologies.* Working Paper 16082. National Bureau of Economic Research. https://doi.org/10.3386/w16082.

Acemoglu, Daron, and Pascual Restrepo. 2018. *Artificial Intelligence, Automation, and Work.* Working Paper 24196. https://doi.org/10.3386/w24196.

Aghion, Philippe, Antonin Bergeaud, Timo Boppart, Peter J. Klenow, and Huiyu Li. 2019a. *A Theory of Falling Growth and Rising Rents.* Working Paper 26448. National Bureau of Economic Research. https://doi.org/10.3386/w26448.

———. 2019b. "Missing Growth from Creative Destruction." *American Economic Review* 109 (8): 2795–2822. https://doi.org/10.1257/aer.20171745.

Akcigit, Ufuk, and Sina T. Ates. 2019. *What Happened to U.S. Business Dynamism?* Working Paper 25756. National Bureau of Economic Research. https://doi.org/10.3386/w25756.

Antras, Pol. 2004. "Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution." *The B.E. Journal of Macroeconomics* 4 (1). https://doi.org/10.2202/1534-6005.1161.

Autor, David, David Dorn, Lawrence F. Katz, Christina Patterson, and John Van Reenen. 2020. "The Fall of the Labor Share and the Rise of Superstar Firms." *The Quarterly Journal of Economics* 135 (2): 645–709. https://doi.org/10.1093/qje/qjaa004.

Autor, David, and Anna Salomons. 2018. *Is Automation Labor-Displacing? Productivity Growth, Employment, and the Labor Share.* Working Paper 24871. National Bureau of Economic Research. https://doi.org/10.3386/w24871.

Baldwin, Carliss Y., and Kim B. Clark. 1995. "Sun Wars: Competition Within A Modular Cluster, 1985-1990." Division of Research, Harvard Business School.

———. 2000. "Design Rules, Volume 1: The Power of Modularity," The MIT Press. https://doi.org/10.7551/mitpress/2366.001.0001.

Barro, Robert J., and Jong Wha Lee. 2013. "A New Data Set of Educational attainment in the World, 1950–2010." *Journal of Development Economics* 104:184–198. https://doi.org/10.1016/j.jdeveco.2012.10.001.

Bernstein, Jeffrey I., and M. Ishaq Nadiri. 1988. "Interindustry R&D Spillovers, Rates of Return, and Production in High-Tech Industries." *American Economic Review* 78 (2): 429–434. http://www.jstor.org/stable/1818163.

Bivens, Josh, Elise Gould, E. Mishel, and Heidi Shierholz. 2014. *Raising America's Pay.* Briefing Paper 378. Economic Policy Institute. https://www.epi.org/publication/raising-americas-pay/.

Bloom, Nicholas, Charles I. Jones, John Van Reenen, and Michael Webb. 2020. "Are Ideas Getting Harder to Find?" *American Economic Review* 110 (4): 1104–44. https://doi.org/10.1257/aer.20180338.

Byrne, David, John Fernald, and Marshall Reinsdorf. 2016. "Does the United States Have a Productivity Slowdown or a Measurement Problem?" *Finance and Economics Discussion Series* 2016:1–74. https://doi.org/10.17016/FEDS.2016.017.

De Loecker, Jan, Jan Eeckhout, and Gabriel Unger. 2020. "The Rise of Market Power and the Macroeconomic Implications." *The Quarterly Journal of Economics* 135 (2): 561–644. https://doi.org/10.1093/qje/qjz041.

Decker, Ryan A., John Haltiwanger, Ron S. Jarmin, and Javier Miranda. 2016. "Declining Business Dynamism: What We Know and The Way Forward." *American Economic Review* 106 (5): 203–07. https://doi.org/10.1257/aer.p20161050.

Doms, Mark E., Wendy E. Dunn, Stephen D. Oliner, and Daniel E. Sichel. 2004. "How Fast Do Personal Computers Depreciate? Concepts and New Estimates." *Tax Policy and The Economy* 18:37–79. https://doi.org/10.1086/tpe.18.20061886.

Dube, Arindrajit, Jeff Jacobs, Suresh Naidu, and Siddharth Suri. 2020. "Monopsony in Online Labor Markets." *American Economic Review: Insights* 2 (1): 33–46. https://doi.org/10.1257/aeri.20180150.

Elsby, Michael W.L., Bart Hobijn, and Ayşegül Şahin. 2013. "The Decline of the U.S. Labor Share." *Brookings Papers on Economic Activity* 2013 (2): 1–63. https://doi.org/10.1353/eca.2013.0016.

Farhi, Emmanuel, and François Gourio. 2018. "Accounting for Macro-Finance Trends: Market power, Intangibles, and Risk Premia." *Brookings Papers on Economic Activity,* 147–223. https://www.jstor.org/stable/26743876.

Feldstein, Martin. 2017. "Underestimating the Real Growth of GDP, Personal Income, and Productivity." *Journal of Economic Perspectives* 31 (2): 145–64. https://doi.org/10.1257/jep.31.2.145.

Fernald, John G. 2015. "Productivity and Potential Output Before, During, and After the Great Recession." *NBER Macroeconomics Annual* 29 (1): 1–51. https://doi.org/10.1086/680580.

Fernald, John G., Robert E. Hall, James H. Stock, and Mark W. Watson. 2017. *The Disappointing Recovery of Output After 2009.* Working Paper 23543. National Bureau of Economic Research. https://doi.org/10.3386/w23543.

Fernald, John G., and Charles I. Jones. 2014. "The Future of U.S. Economic Growth." *American Economic Review* 104 (5): 44–49. https://doi.org/10.1257/aer.104.5.44.

Geske, Michael J., Valerie A. Ramey, and Matthew D. Shapiro. 2007. "Why Do Computers Depreciate?" In *Hard-to-Measure Goods and Services: Essays in Honor of Zvi Griliches,* 121–150. University of Chicago Press. https://doi.org/10.7208/chicago/9780226044507.001.0001.

Gordon, Robert J. 2016. "The Rise and Fall of American Growth: The U.S. Standard of Living since the Civil War." Princeton University Press. https://doi.org/doi:10.1515/9781400873302.

Growiec, Jakub, Peter McAdam, and Jakub Mućk. 2018. "Endogenous Labor Share Cycles: Theory and Evidence." *Journal of Economic Dynamics and Control* 87:74–93. https://doi.org/10.1016/j.jedc.2017.11.007.

Gutiérrez, Germán, and Thomas Philippon. 2016. *Investment-less Growth: An Empirical Investigation.* Working Paper 22897. National Bureau of Economic Research. https://doi.org/10.3386/w22897.

Hall, Robert E. 2009. "Reconciling Cyclical Movements in The Marginal Value of Time and The Marginal Product of Labor." *Journal of Political Economy* 117 (2): 281–323. https://doi.org/10.1086/599022.

Hulten, Charles R. 1991. "The Measurement of Capital." In *Fifty Years of Economic Measurement: The Jubilee of the Conference on Research in Income and Wealth,* 119–158. University of Chicago Press. https://EconPapers.repec.org/RePEc:nbr:nberch:5974.

Jones, Charles I., and John C. Williams. 2000. "Too Much of a Good Thing? The Economics of Investment in R&D." *Journal of Economic Growth* 5 (1): 65–85. https://doi.org/10.1023/A:1009826304308.

Jorgenson, Dale W., Mun S. Ho, and Jon D. Samuels. 2014. "Long-term Estimates of U.S. Productivity and Growth." In *Prepared for Presentation at the Third World KLEMS Conference: Growth and Stagnation in the World Economy.*

Kaldor, Nicholas. 1961. "Capital Accumulation and Economic Growth." In *The Theory of Capital,* 177–222. Palgrave Macmillan. https://doi.org/10.1007/978-1-349-08452-4_10.

Karabarbounis, Loukas, and Brent Neiman. 2014. "The Global Decline of the Labor Share." *The Quarterly Journal of Economics* 129 (1): 61–103. https://doi.org/10.1093/qje/qjt032.

Klump, Rainer, Peter McAdam, and Alpo Willman. 2007. "Factor Substitution and Factor-Augmenting Technical Progress in the United States: A Normalized Supply-Side System Approach." *The Review of Economics and Statistics* 89 (1): 183–192. https://doi.org/10.1162/rest.89.1.183.

Kozmetsky, George, and Piyu Yue. 2005. "The Economic Transformation of the United States, 1950-2000: Focusing on the Technological Revolution, the Service Sector Expansion, and the Cultural, Ideological, and Demographic Changes." Purdue University Press.

Krusell, Per, Lee E. Ohanian, José-Víctor Ríos-Rull, and Giovanni L. Violante. 2000. "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis." *Econometrica* 68 (5): 1029–1053. https://doi.org/10.1016/j.red.2007.09.001.

Lawrence, Robert Z. 2015. *Recent Declines in Labor's Share in U.S. Income: A Preliminary Neoclassical Account.* Working Paper 21296. National Bureau of Economic Research. https://doi.org/10.3386/w21296.

León-Ledesma, Miguel A., Peter McAdam, and Alpo Willman. 2010. "Identifying the Elasticity of Substitution with Biased Technical Change." *American Economic Review* 100 (4): 1330–57. https://doi.org/10.1257/aer.100.4.1330.

Lindenberg, Eric B., and Stephen A. Ross. 1981. "Tobin's Q Ratio and Industrial Organization." *Journal of Business,* 1–32. http://www.jstor.org/stable/2352631.

Liu, Ernest, Atif Mian, and Amir Sufi. 2022. "Low Interest Rates, Market Power, and Productivity Growth." *Econometrica* 90 (1): 193–221. https://doi.org/https://doi.org/10.3982/ECTA17408.

Oliner, Stephen D. 2007. "Constant-Quality Price Change, Depreciation, and Retirement of Mainframe Computers." In *Price Measurements and Their Uses,* 19–62. University of Chicago Press. https://doi.org/doi:10.7208/9780226257327-003.

Oliner, Stephen D., Daniel E. Sichel, Jack E. Triplett, and Robert J. Gordon. 1994. "Computers and Output Growth Revisited: How Big is the Puzzle?" *Brookings Papers on Economic Activity* 1994 (2): 273–334. https://doi.org/10.2307/2534658.

Patrikalakis, Nicholas M., and Takashi Maekawa. 2002. "Geodesics: Initial Approximation." In *Shape Interrogation for Computer Aided Design and Manufacturing.* Available in http://web.mit.edu/hyperbook/Patrikalakis-Maekawa-Cho/node2.html. Springer Berlin Heidelberg.

Piketty, Thomas, and Gabriel Zucman. 2014. "Capital is Back: Wealth-Income Ratios in Rich Countries 1700–2010." *The Quarterly Journal of Economics* 129 (3): 1255–1310. https://doi.org/10.1093/qje/qju018.

Ravn, Morten O., and Harald Uhlig. 2002. "On Adjusting the Hodrick-Prescott Filter for the Frequency of Observations." *The Review of Economics and Statistics* 84 (2): 371–376. https://doi.org/10.1162/003465302317411604.

Rognlie, Matthew. 2016. "Deciphering the Fall and Rise in the Net Capital Share: Accumulation or Scarcity?" *Brookings Papers on Economic Activity* 2015 (1): 1–69. https://doi.org/10.1353/eca.2016.0002.

Shiller, Robert. 2015. "Irrational Exuberance." 3rd ed. Princeton University Press. https://doi.org/10.2307/j.ctt1287kz5.

Syverson, Chad. 2017. "Challenges to Mismeasurement Explanations for the U.S. Productivity Slowdown." *Journal of Economic Perspectives* 31 (2): 165–86. https://doi.org/10.1257/jep.31.2.165.

Trimborn, Timo, Karl-Josef Koch, and Thomas M. Steger. 2008. "Multidimensional Transitional Dynamics: A Simple Numerical Procedure." *Macroeconomic Dynamics* 12 (3): 301–319. https://doi.org/10.1017/S1365100507070034.

[dataset] U.S. Bureau of Economic Analysis. 2019. "National Product and Income Account (NIPA) Tables." Data retrieved from https://apps.bea.gov/iTable/index_FA.cfm.

[dataset] U.S. Bureau of Labor statistics. 2019. "Labor Productivity and Costs Measures." Data retrieved from https://www.bls.gov/lpc/#tables.

Uzawa, Hirofumi. 1961. "Neutral Inventions and the Stability of Growth Equilibrium." *The Review of Economic Studies* 28 (2): 117–124. https://doi.org/10.2307/2295709.

Appendices

A. Omitted Derivations and Proofs

A.1. Monopoly Prices

Final good producers faces following problem (for labor-intensive bundle):

$$\max_{\{y_l(\nu)\}_{\nu=0}^{\nu_h}} P_L \left[\int_0^{\nu_h} y_l(\nu)\, d\nu\right]^{\frac{1}{}} - \int_0^{\nu_h} p_l(\nu) y_l(\nu)\, d\nu \tag{A.1}$$

Then, isoelastic demand for $y_l(\nu)$ is

$$p_l(\nu) = P_L \left[\frac{y_l(\nu)}{Y_L}\right]^{\frac{1}{}} \tag{A.2}$$

I take the power of $\frac{}{1}$ and integrate over ν,

$$\int_0^{\nu_h} p_l(\nu)^{\frac{}{1}}\, d\nu = P_L^{\frac{}{1}} Y_L \int_0^{\nu_h} y_l(\nu)\, d\nu \tag{A.3}$$

where $\int_0^{\nu_h} y_l(\nu)\, d\nu = Y_L$, I get ideal price index as follows:

$$P_L = \left[\int_0^{\nu_h} p_l(\nu)^{\frac{}{1}}\, d\nu\right]^{\frac{}{1}} \tag{A.4}$$

A monopolist's profit maximization problem is

$$\max_{p_l(\nu)} y_l(\nu)\left[p_l(\nu) - w\right] \tag{A.5}$$

Using (A.2) and (A.4), I can write (A.5) as

$$\max_{p_l(\nu)} Y_L \frac{p_l(\nu)^{\frac{1}{1}}}{\int_0^{\nu_h} p_l(\nu)^{\frac{}{1}}\, d\nu}\left[p_l(\nu) - w\right] \tag{A.6}$$

Taking logarithm of (A.6) of firm ν^{th}, first order condition (FOC) yields

$$\frac{1}{1}\frac{1}{p_l()} + \frac{1}{1} \ \mathcal{P}^h_{=0}\frac{1}{p_l()^{-1}d} \ p_l()^{-\frac{1}{1}} + \frac{1}{p_l()} \ \frac{1}{w} = 0 \tag{A.7}$$

FOC is same for all monopolists, $p_l() = p_l$, then I have

$$\mathcal{X}^h_{=0} p_l()^{-1}d = hp_l^{-1} \tag{A.8}$$

Rearranging equation (A.7) using (A.8), now $p_l()$ can be defined by $p_l() = \frac{wh+w}{h\ 1}$. Taking limit, price in labor-intensive intermediate goods sector (in the same fashion price in the capital-intensive intermediate sector) is defined as a constant over marginal cost,

$$\lim_{h! \ 1} p_l() = \frac{w}{} \quad and \quad \lim_{z! \ 1} p_k() = \frac{r}{} \tag{A.9}$$

A.2.   Value of Monopolists

In an equilibrium value functions of monopolists satisfy corresponding Hamilton-Jacobi-Bellman (HJB) equations. For labor-intensive sector HJB equation is $rV_l \quad V_l = \quad _l$ while for capital-intensive sector HJB equation is $(r + \ )V_k \quad V_k = \quad _k$.

The free entry condition is given by $V_l b_h h = V_k b_z z = w$ and the wage rate is defined by the equation $w = h^{1-} P_L$. By differentiating $V_l b_h h = \ h^{1-} P_L$, I obtain

$$\frac{V_l}{V_l} = \frac{1 \quad 2}{} \frac{h}{h} \tag{A.10}$$

Let $\frac{h}{h} \quad \frac{}{1} g$ where g denotes productivity growth. Substituting (A.10) into HJB equation and using $_l = \frac{1-}{} \frac{wL}{h}$, I obtain the value function of labor-intensive monopolists,

$$V_l = \frac{1}{r} \frac{wL=h}{\frac{1\ 2}{1} g} \tag{A.11}$$

Now I will define the value function of capital-intensive monopolist. Differentiating $V_k b_z z = \ h^{1-} P_L$, I get

$$\frac{V_k}{V_k} = \frac{1}{} \frac{h}{h} \tag{A.12}$$

Substituting this into HJB equation and using $\kappa = \frac{1-\alpha}{\alpha}\frac{rK}{z}$, the value function of capital-intensive monopolists is defined by

$$V_k = \frac{1}{r+\delta}\frac{rK=z}{\phi} \tag{A.13}$$

A.3. Long-run Productivity Growth and Relative Capital Income Share

From technology equations in (6), where $\frac{\dot{h}}{h} \equiv \frac{1}{1-\beta}g$, long-run employment shares in R&D sectors are

$$L_z = \frac{\phi}{b_z} \quad \text{and} \quad L_h = \frac{1}{1-\beta}\frac{g}{b_h} \tag{A.14}$$

Substituting A.11 into $b_h h V_l(t) = w$ and using Euler equation, $g+\rho = r$, I obtain long-run employment in production sector,

$$L = g\left[\frac{(1-\alpha)(1-2\alpha)}{b_h(1-\alpha)^2} + \frac{(1-\alpha)}{b_h(1-\alpha)^2}\right] \tag{A.15}$$

Recall that clearing condition in labor market is $L_h + L_z + L = 1$. Substituting (A.14) and (A.15) into clearing condition reveals long-run economic (or productivity) growth,

$$g = \frac{1}{\phi}\frac{b_h-\frac{b_h}{b_z}-\frac{1}{1-\beta}}{\tilde{\psi}+1} \tag{A.16}$$

where $\tilde{\psi} \equiv \frac{1-2\alpha}{1-\alpha}$. Let me plug (A.11) and (A.13) into free entry condition (21) to get the following equation:

$$\frac{1}{\phi}\frac{b_z r K}{r+\delta-g} = \frac{1}{\phi}\frac{b_h w L}{r-\tilde{\psi}g} \tag{A.17}$$

From (A.17), I get long-run relative income share of capital,

$$\kappa_L = \frac{r K}{w L} = \frac{g(\eta-1)+\rho+\delta}{g(\eta-\tilde{\psi})+\rho} \tag{A.18}$$

where $\eta \equiv \frac{1}{1-\beta}\frac{b_h}{b_z}$. Note that $\frac{P_K Y_K}{P_L Y_L} = \frac{rk=}{wL=}$; total profits acquired in the labor- and capital-intensive sectors are included in labor and capital incomes, respectively.

iii

A.4. Redefining Rate of Interest

Using linear homogeneity, I can write CES price index referring to the price of final good as follows:

$$1 = \left[\theta^{\frac{1}{\gamma}} P_L^{1-\gamma} + (1-\theta)^{\frac{1}{\gamma}} P_K^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \tag{A.19}$$

where final good is numeraire and its price can be normalized to 1. Using the definition of relative price of capital-intensive aggregate goods (17), I get P_K in terms of efficient capital per efficient labor, , such as

$$P_K = \theta^{\frac{1}{\gamma}} \left[\left(\frac{1}{\theta} \right)^{\frac{1}{\gamma}} + 1 \right]^{\frac{1}{1-\gamma}} \tag{A.20}$$

Since capital is paid its marginal product, $r = z^{1-\gamma} P_K$. From (A.20) the rental rate of capital (rate of interest) is

$$r = \left[(1-\theta) z^{\frac{1-\gamma}{\gamma}} + (1-\theta) \right]^{\frac{1}{1-\gamma}} \tag{A.21}$$

A.5. Source of Wage-Productivity Gap

From (A.19), the price of labor-intensive aggregate goods is

$$P_L = \left[\theta^{\frac{1}{\gamma}} + (1-\theta)^{\frac{1}{\gamma}} \frac{1}{1} \right]^{\frac{1}{1-\gamma}} \tag{A.22}$$

Labor is paid its marginal product, $w = h^{1-\gamma} P_L$. Now let me take differentiation of this equation, $\frac{\dot{w}}{w} = \frac{1-\gamma \dot{h}}{h} + \frac{\dot{P_L}}{P_L}$. Also differentiating (A.22), I obtain

$$\frac{\dot{w}}{w} = \frac{1-\gamma \dot{h}}{h} + \frac{(1-\theta)(1-\gamma)}{+(1-\theta)} \tag{A.23}$$

I define labor productivity growth as $\frac{\dot{Y_L}}{Y_L} \quad \frac{\dot{L}}{L} = \frac{1-\gamma \dot{h}}{h}$. So, wage growth has two sources: productivity growth and change in efficient capital-labor ratio, . Along balanced-growth path, $\dot{} = 0$ must hold, then wage growth is only determined by productivity growth (economic growth). In case of a positive permanent shock, the economy moves away

iv

from its initial BGP and efficient capital-labor ratio declines. Hence, wage growth falls behind the productivity growth until the economy reaches a new BGP.

B. Computational Approach

B.1. Transformed System

The dynamic behaviour of the model is defined by a system of differential equations, in terms of growth rates, augmented with equilibrium conditions:

$$\frac{\dot{c}}{c} = \frac{\dot{C}}{C} + \frac{1}{}\frac{\dot{Z}}{Z} \quad \frac{1}{}\frac{\dot{h}}{h} \tag{B.1}$$

$$\dot{-} = \frac{\dot{K}}{K} + \frac{1}{}\frac{\dot{Z}}{Z} \quad \frac{1}{}\frac{\dot{h}}{h} \tag{B.2}$$

$$\frac{\dot{Z}}{Z} = b_z L_z \tag{B.3}$$

$$V_l b_h h = V_k b_z Z \tag{B.4}$$

$$L_h + L_z = 1 \quad L \tag{B.5}$$

Note that c is normalized consumption such that $c \quad \frac{ZC}{HL}$ and is normalized capital-labor ratio, i.e., efficient capital-labor ratio, such that $\frac{ZK}{HL}$, where $H \quad h^{1}$ and $Z \quad z^{1}$.

BGP requires c and to be constant so that H, K and C grow at a common rate while Z and L remain unchanged. (B.4) implies that R&D firms are indifferent between investing in labor-and capital-augmenting technologies. Thus, (B.4) and equilibrium condition in labor market (B.5) determine labor allocation across production sector, labor-augmenting and capital-augmenting R&D sectors.

Let me redefine law of motion of normalized consumption plugging (A.21) into the Euler Equation $\frac{\dot{C}}{C} = \frac{r}{}$ and using evolutions of labor- and capital-augmenting technologies, $\frac{\dot{h}}{h} = (b_h L_h)$ and $\frac{\dot{z}}{z} = (b_z L_z \quad)$.

$$\frac{\dot{c}}{c} = \frac{1}{} \quad (1 \quad) z^{1} \quad + (1 \quad)^{1} \\ + \frac{1}{}(b_z L_z \quad) \quad \frac{1}{} b_h L_h \tag{B.6}$$

I also redefine law of motion of   as follows:

$$\frac{\dot{-}}{-} = \frac{z^{\frac{1}{-}}(\ + (1\ \ )\ \ )^{\frac{1}{-}}\ c}{-} + \frac{1}{-}(b_z L_z\ \ )^{\frac{1}{-}} \ \ \frac{1}{-} b_h L_h \tag{B.7}$$

From (A.11), (A.13) (A.21), (B.4), where $-\frac{rK}{wL} = \frac{1\ \ }{-}$, in a BGP following equation must hold;

$$0 = \frac{1\ \ }{-} \ \ \ \ \frac{(1\ \ )\ b_h}{(1\ \ )\ b_z}\ \frac{(1\ \ )z^{\frac{1}{-}} \ \ \ + (1\ \ )^{\frac{1}{-}} + \ \ \frac{1}{-} b_h L_h}{(1\ \ )z^{\frac{1}{-}} \ \ \ + (1\ \ )^{\frac{1}{-}} \ \ \frac{1\ 2}{-} b_h L_h} \tag{B.8}$$

L is implicit in all equations. I do not need it specifically, but in terms of L_h such that $L = \frac{}{(1\ \ )b_h} r^{\ \ \frac{1\ 2}{1}} L_h$, where r is defined by (A.21). Together with this, from (B.5) the following equation must also hold in a BGP;

$$0 = \frac{}{1\ \ } L_h + L_z + \frac{(1\ \ )}{b_h(1\ \ )} z^{\frac{1}{-}} \ \ \ \ \ + (1\ \ )^{\frac{1}{-}} \ \ 1 \tag{B.9}$$

My aim is to numerically solve the three-dimensional system of differential equations where   and z are state variables and c is control variable, taking into account interior conditions (B.8) and (B.9) which must hold around BGP. This system admits one positive and two negative roots, which ensure the saddle-path stability.

B.2.   Relaxation Procedure

The vector of components of ordinary differential and static equations is defined by **x** $(c;\ ; z; L_h; L_z)$. The dynamic system which governs the evolution of the economy is

$$(\dot{-}; \dot{z}; \dot{c}) = f(\mathbf{x}) \tag{B.10}$$

$$0 = g(\mathbf{x}) \tag{B.11}$$

Let   refer to a rescaled time parameter, $= \frac{t}{1+\ t}$. I restrict the time interval of the simulation $T = f\ _1;\ _2; ::::;\ _m g$ which is a mesh of m points. I set $= 0.04$ and $T = 200$.

vi

Taking finite differences on the mesh, I approximate the governing equations using the following formula:

$$\frac{\mathbf{x}_k - \mathbf{x}_{k-1}}{\tau_k - \tau_{k-1}} = f\left(\frac{\mathbf{x}_k + \mathbf{x}_{k-1}}{2}\right) \quad \text{for} \quad k = 2, 3, \ldots, m \tag{B.12}$$

with boundary conditions, where the number of initial conditions is to be denoted by n_1, and the number of final conditions by n_2. Also, n_3 does denote the codimension of the manifold supplemented by $g(\tau, x) = 0$. In this study, $n_1 = 2$, $n_2 = 1$, and $n_3 = 2$. So, the dimension of the differential equations is $n_1 + n_2 = 3$, denoted by n. Finally, let me define $ñ = n_2 + n_3$.

Equation (B.12) reveals a system of $(m-1)n$ differential equations with mn unknowns; $\mathbf{x}_k = (x_1, x_2, \ldots, x_n)_k^T$ for $k = 1, \ldots, m$. Then, $(m-1)n$ errors are

$$\varepsilon_k = (\varepsilon_{1,k}, \varepsilon_{2,k}, \ldots, \varepsilon_{n,k})^T = \frac{\mathbf{x}_k - \mathbf{x}_{k-1}}{\tau_k - \tau_{k-1}} - f\left(\frac{\mathbf{x}_k + \mathbf{x}_{k-1}}{2}\right) \tag{B.13}$$

where $k = 2, 3, \ldots, m$. Note that $\mathbf{x}_1$ has n_1 known components while $\mathbf{x}_m$ and $\mathbf{x}_k$ respectively have n and $ñ$ known components so that

$$\varepsilon_1 = (\varepsilon_{1,1}, \varepsilon_{2,1}, \ldots, \varepsilon_{n_1,1})^T = 0 \tag{B.14}$$

$$\varepsilon_k = (\varepsilon_{1,k}, \varepsilon_{2,k}, \ldots, \varepsilon_{n,k})^T = 0 \tag{B.15}$$

$$\varepsilon_{m+1} = (\varepsilon_{1,m+1}, \varepsilon_{2,m+1}, \ldots, \varepsilon_{ñ,m+1})^T = 0 \tag{B.16}$$

Hence, I have mn differential equations, $\varepsilon = (\varepsilon_1^T, \varepsilon_2^T, \ldots, \varepsilon_{m+1}^T)^T$. If a good initial approximation is provided to guess final steady-state values, $\mathbf{x}^{(0)} = (\mathbf{x}_1^T, \mathbf{x}_2^T, \ldots, \mathbf{x}_m^T)^T$, the system of differential equations can be solved iteratively using Newton Method.

$$\mathbf{x}^{(i+1)} = \mathbf{x}^{(i)} + \Delta\mathbf{x}^{(i)} \tag{B.17}$$

$$\|\mathbf{J}^{(i)}\| \Delta\mathbf{x}^{(i)} = -\varepsilon^{(i)} \tag{B.18}$$

where i denotes i^{th} iteration and $\mathbf{J}^{(i)}$ is mn by mn matrix of $\varepsilon^{(i)}$ with respect to $\mathbf{x}^{(i)}$.
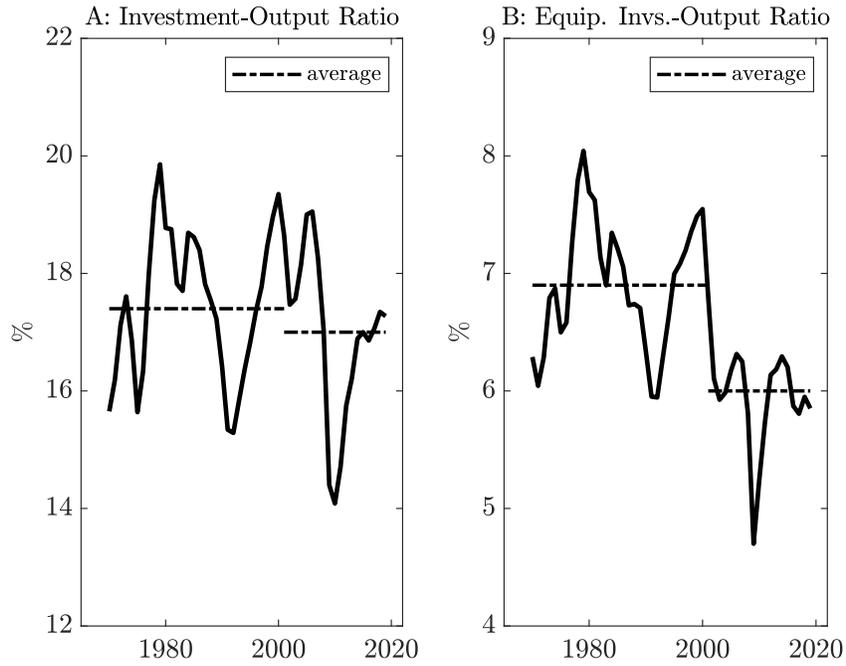
C. Additional Figures

Figure C.1: Investment Shares of GDP (U.S. total industry, 1970-2019)

Source: U.S. Bureau of Economic Analysis, National Product and Income Account (NIPA) Tables

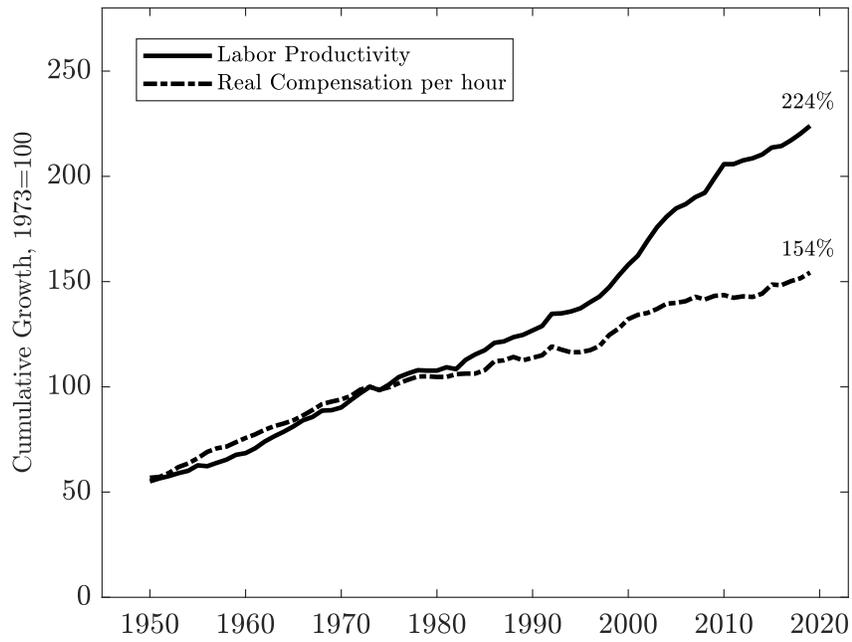**Figure C.2:** Wage-Productivity Gap (U.S. nonfarm business, 1950-2019)

Source: U.S. Bureau of Labor Statistics, Labor Productivity and Costs Measures